

An Energy-Focused Order Reduction Envelope Model for Parallel-Parallel Circuits in Inductive Power Transfer

Tomonobu Saikawa

Faculty of Science and Engineering
Tokyo University of Science
Chiba, Japan
saikawa.tomonobu25@gmail.com

Takehiro Imura

Faculty of Science and Engineering
Tokyo University of Science
Chiba, Japan

Yoichi Hori

Faculty of Science and Engineering
Tokyo University of Science
Chiba, Japan

Abstract— In magnetic resonance-based wireless power transmission systems, the large number of components results in high-order calculations, making it difficult to analytically determine transient waveforms. Therefore, reducing the order is necessary for control applications or for analyzing transients. This paper proposes reducing the order in a P-P circuit. Inspired by the Energy Balanced Module theory (EBM method) used in prior research on S-S resonant circuits, we employ a proposed approach that treats energy as a mediating variable. By focusing on the current amplitude under specific conditions, we successfully developed an order-reduction envelope model. This enables a significant reduction in the order of the P-P circuit, from fourth-order to second-order. The envelope model developed here is robust to variations in load and coupling coefficients, making it promising for applications such as control. Under optimal load conditions, the steady-state relative error was found to be small: 1.3% for the transmission-side current and 1.3% for the reception-side current.

Keywords— *Wireless Power Transfer, Magnetic resonant coupling, transient analysis, Parallel-parallel resonant circuit*

I. INTRODUCTION

Wireless power transfer (WPT) transmits electrical power through magnetic or electric field coupling. During this process, transient phenomena can induce overcurrent and overvoltage, which must be suppressed because they degrade device lifetime. On the other hand, resonant circuits in magnetic coupling WPT systems consist of many components and exhibit high-order characteristics, making analytical solutions difficult to obtain [1][2][3][4].

The AC current source P-P circuit used in this paper has advantages over the AC current source S-S circuit, such as higher output power and a larger optimal load [7]. Therefore, it is necessary to perform transient analysis for the P-P circuit as well. As prior research, the Energy Balanced Module theory (EBM method) has been developed for S-S resonant circuits. In this theory, there exists a timing at which the phase difference between the current and the capacitor voltage becomes 90°, and the phase difference between the transmitter-side and receiver-side currents coincides at their maximum amplitudes, enabling

smooth theoretical development. In contrast, in the P-P circuit, the phase difference between the current and the capacitor voltage does not reach 90° on either the transmission or reception side. Furthermore, the phase difference between the transmission-side and reception-side currents does not coincide at their maximum amplitudes. Consequently, the EBM method cannot be directly applied to the P-P circuit. Therefore, this paper is inspired by the Energy Balanced Module theory (EBM method) previously applied to S-S resonant circuits and proposes that, by focusing on energy, an order-reduced envelope model can be constructed even for the more complex P-P circuit. Furthermore, the proposed method investigates and takes into account a key characteristic of the P-P circuit, namely that the coupling coefficient appears in the resonance condition. This paper is organized as follows. Section 2 reviews the prior research. Section 3 describes the proposed method. Section 4 presents simulation and experimental results using actual hardware. Finally, Section 5 concludes this paper.

II. PREVIOUS RESEARCH

A. Overview

The previous study proposed a method for order reduction of transient currents and voltages by focusing on the total energy of the circuit[3].

B. Theory

In Fig. 1, state equations for the current and voltage are formulated. By assuming that the internal resistance of the inductor is sufficiently small and can be neglected, prior research has shown that both the current and voltage exhibit oscillatory behavior [3]. On the other hand, the circuit energy can be expressed as a function including the current and voltage [3]. By substituting Eqs. (2) and (3) into Eq. (4), Eq. (5) is obtained [3].

$$\begin{bmatrix} i'_L \\ v'_c \end{bmatrix} = \begin{bmatrix} \frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_g \quad (1)$$

$$i_L \approx U_g \frac{1}{R} \cos(\omega t) \left(1 - e^{-\frac{R}{2L}t}\right) \quad (2)$$

$$v_c \approx U_g \frac{\omega L}{R} \sin(\omega t) \left(1 - e^{-\frac{R}{2L}t}\right) \quad (3)$$

$$W_{LC} = \frac{1}{2} L i_L^2 + \frac{1}{2} C v_c^2 \quad (4)$$

$$W_{LC} = \frac{L U_g^2}{2 R^2} \left(1 + e^{\frac{R}{L}} - 2e^{-\frac{R}{2L}}\right) \quad (5)$$

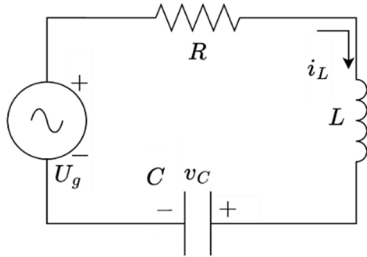


Fig.1 LC resonant tank. [3]

Therefore, the circuit energy does not oscillate and gradually converges to a constant value. Accordingly, when the circuit current reaches its maximum, the voltage becomes zero, and when the voltage reaches its maximum, the current becomes zero. Therefore, it is unnecessary to consider the phase, and only the condition in which the current is maximum needs to be considered. In addition, the circuit energy can be evaluated by considering either the inductor or the capacitor alone, and the value of the other component can be determined from the circuit energy [3]. Next, for the S-S circuit, the current is calculated in terms of its amplitude, and can be expressed as shown in (6) and (7) [3].

$$W_{LC1} = \frac{1}{2} L I_L^2 \quad (6)$$

$$W_{LC2} = \frac{1}{2} L I_L^2 \quad (7)$$

Furthermore, the variation in circuit energy is caused by the energy supplied from the power source, the losses in the resistors, and the power exchanged between the transmission and reception coils, and can be expressed by (8) and (9) [3].

$$W'_{LC1} = L_1 I_{L1} I'_{L1} = \frac{1}{2} U_g I_{L1} - \frac{1}{2} I_{L1}^2 R_1 - \frac{1}{2} \omega M I_{L1} I_{L2} \quad (8)$$

$$W'_{LC2} = L_2 I_{L2} I'_{L2} = \frac{1}{2} \omega M I_{L1} I_{L2} - \frac{1}{2} I_{L2}^2 (R_2 + R_L) \quad (9)$$

Equations (8) and (9) can be rewritten in state-space form as (10) [3].

$$\begin{bmatrix} I'_{L1} \\ I'_{L2} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{2L_1} & -\frac{\omega M}{2L_2} \\ \frac{\omega M}{2L_2} & -\frac{(R_2 + R_L)}{2L_1} \end{bmatrix} \begin{bmatrix} I_{L1} \\ I_{L2} \end{bmatrix} + \begin{bmatrix} \frac{1}{2L_1} \\ 0 \end{bmatrix} U_g \quad (10)$$

C. Simulation Results

When the proposed method (EBM method) is applied to the S-S circuit shown in Fig. 2, the comparison with the simulation results is presented in Fig. 3.

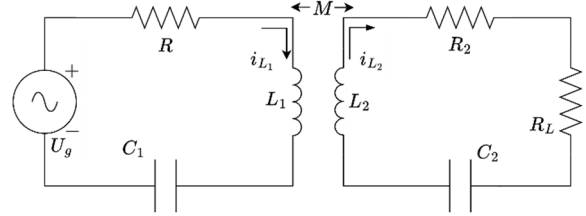
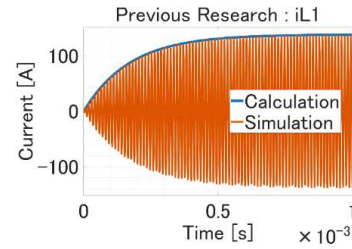
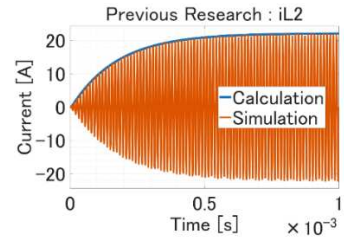


Fig.2 Two coupled LC resonant tanks. [3]



(a)



(b)

Fig. 3 Time-domain solutions of two coupled LC resonant tanks from the instantaneous model and EBM[3].

Therefore, the envelope model using the EBM method shows good agreement with the waveform and amplitude.

III. PROPOSED METHODOLOGY

A. Overview

We considered that the EBM method proposed in prior research can be extended to the P-P circuit in IPT systems. In the S-S circuit presented in the prior research, the phase difference between the transmission-side current and the voltage across the transmission-side capacitor is 90° , and the same relationship holds for the reception side. Since these conditions occur simultaneously, the principle of order reduction can be derived mathematically. However, such conditions do not arise in the P-P circuit. Therefore, by focusing on the points where the transmission-side and reception-side currents reach their maximum values and utilizing the fact that the energies stored in the inductor and capacitor are equal at those points, the order-reduced envelope model was established by doubling the inductor energy.

B. Calculation

First, the P-P resonant circuit is shown in Fig. 4. Since the current source and resonant capacitor are connected in parallel in Fig. 4, this part can be transformed into an equivalent voltage source, resulting in the circuit shown in Fig. 5.

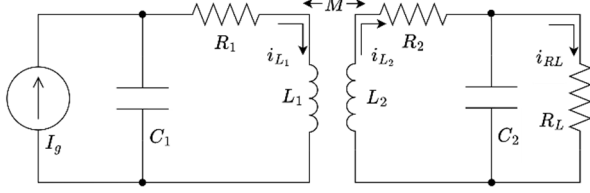


Fig. 4 P-P circuit

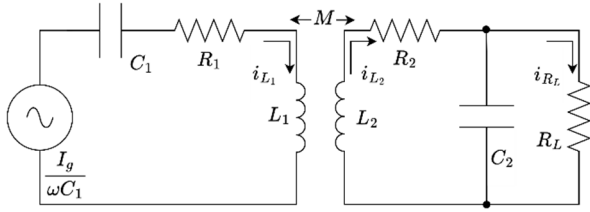


Fig. 5 P-P circuit (After conversion to a voltage source)

By applying the equivalent transformation to the transmission side, the same relationship as in (5) can be applied to the P-type transmission side. Therefore, there are cases in which the capacitor current becomes zero when the current reaches its maximum. Furthermore, focusing on the P-type receiving side, the load resistance connected in parallel with the capacitor has the same voltage as the capacitor. Therefore, when the current flowing through the inductor reaches its maximum, the voltage stored in the capacitor becomes zero. In this case, the capacitor can be regarded as a short circuit, and thus no current flows through the load resistance. Consequently, similarly to the S-type case, the capacitor current can become zero when the current reaches its maximum. In addition, the energy variation in the receiving-side circuit consists only of the energy received from the transmission side and the energy dissipated in the resistor. Therefore, the circuit energy does not vary significantly during one cycle. Accordingly, the energy in both the transmission-side and receiving-side circuits remains approximately constant. Here, the transmission-side circuit energy is defined as W_{P1} , and the receiving-side circuit energy is defined as W_{P2} . Furthermore, since a sinusoidal power source is used as the input source, there are instances in which the energy stored in L_1 becomes equal to that stored in C_1 . Therefore, the relationship in (11) may hold. From (11), it follows that (12) is obtained.

$$\frac{1}{2} L_1 i_{L1s}^2 = \frac{1}{2} C_1 v_{C1s}^2 \quad (11)$$

$$W_{P1} = \frac{1}{2} L_1 i_{L1s}^2 + \frac{1}{2} C_1 v_{C1s}^2 = L_1 i_{L1s}^2 \quad (12)$$

Next, since the energy within the circuit does not vary significantly, and considering the existence of the condition expressed in (11), (12), the relationships can be written as (13) and (14). Furthermore, in order to consider the amplitude of the

upper envelope, assuming the current to be positive, (14) can be rewritten as (15).

$$W_{P1} = \frac{1}{2} L_1 I_{L1}^2 = L_1 i_{L1s}^2 \quad (13)$$

$$i_{L1s}^2 = \frac{1}{2} I_{L1}^2 \quad (14)$$

$$i_{L1} = \frac{I_{L1}}{\sqrt{2}} = I_{L1} \sin\left(\frac{\pi}{4}\right) \quad (15)$$

Furthermore, due to the gyrator characteristics, the phase difference between the transmission-side current and the reception-side current is 90° . Therefore, the reception-side inductor current corresponding to the condition in (15) can be expressed as (16).

$$i_{L2s} = I_{L2} \left| \sin\left(\frac{\pi}{4} - \frac{\pi}{2}\right) \right| = I_{L2} \sin\left(\frac{\pi}{4}\right) = \frac{I_{L2}}{\sqrt{2}} \quad (16)$$

Moreover, in the receiving-side circuit, similarly to the transmission-side circuit, current is supplied from the transmission side driven by a sinusoidal power source. Therefore, there exist conditions under which the energy stored in L_2 is equal to that stored in C_2 . From (16), it follows that when (11) holds, (17) is satisfied on the receiving side. Accordingly, (18) is obtained.

$$\frac{1}{2} L_2 i_{L2s}^2 = \frac{1}{2} C_2 v_{C2s}^2 \quad (17)$$

$$W_{P2} = \frac{1}{2} L_2 I_{L2}^2 = L_2 i_{L2s}^2 + \frac{1}{2} C_2 v_{C2s}^2 \quad (18)$$

Next, the current and voltage on the receiving side are derived. From Ohm's law, (19) is obtained and can be rearranged into the forms shown in (20) and (21). Finally, by considering W_{P1} , W_{P2} , equations (22) and (23) can be derived from (13) and (18).

$$v_{C2} = R_L i_{RL} \quad (19)$$

$$i_{RL} = \frac{v_{C2}}{R_L} \quad (20)$$

$$v_{C2} = \sqrt{\frac{L_2}{C_2}} i_{L2} \quad (21)$$

$$W_{P1} = \frac{1}{2} L_1 I_{L1}^2 \quad (22)$$

$$W_{P2} = \frac{1}{2} L_2 I_{L2}^2 \quad (23)$$

By considering the time derivatives of (22) and (23), (24) and (25) are obtained.

$$W'_{P1} = L_1 I_{L1} I'_{L1} \quad (24)$$

$$W'_{P2} = L_2 I_{L2} I'_{L2} \quad (25)$$

Considering the variation in energy, (26) and (27) are derived using (28).

$$W'_{P1} = \frac{1}{2} U_g I_{L1} - \frac{1}{2} R_1 I_{L1}^2 - \frac{1}{2} \omega M I_{L1} I_{L2} \quad (26)$$

$$\begin{aligned} W'_{P2} &= \frac{1}{2} \omega M I_{L1} I_{L2} - \frac{1}{2} R_2 I_{L2}^2 - \frac{1}{2} R_L I_{RL}^2 \\ &= \frac{1}{2} \omega M I_{L1} I_{L2} - \frac{1}{2} R_2 I_{L2}^2 - \frac{1}{2} R_L \left(\frac{1}{R_L} \sqrt{\frac{L_2}{C_2}} I_{L2} \right)^2 \end{aligned} \quad (27)$$

Therefore, by treating energy as an intermediate variable and deriving the state equations for the currents from (24), (25), (26), and (27), the state equation is given by (28).

$$\begin{bmatrix} I'_{L1} \\ I'_{L2} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{2L_1} & -\frac{\omega M}{2L_1} \\ \frac{\omega M}{2L_2} & -\frac{R_2}{2L_2} - \frac{1}{2R_L C_2} \end{bmatrix} \begin{bmatrix} I_{L1} \\ I_{L2} \end{bmatrix} + \begin{bmatrix} \frac{1}{2L_1} \\ 0 \end{bmatrix} \frac{I_g}{\omega C_1} \quad (28)$$

IV. SIMULATION AND EXPERIMENTAL RESULTS

A. Simulation and Experimental Conditions

First, the element values used in this study are listed in Table 1. Initially, a comparison was performed under the optimal load condition using the element values in Table 1. The optimal load is defined as the load that maximizes efficiency and is given by (29). Next, simulations were conducted for four additional cases: two cases in which the optimal load was varied while keeping the element values in Table 1 constant, and two cases in which the coupling coefficient k was varied while using the optimal load. In total, five simulation cases were considered. In the experimental setup, an AC voltage source was used instead of an AC current source due to its ease of implementation. Therefore, the circuit in Fig. 5 was obtained by converting the original circuit into an equivalent circuit using an AC voltage source. Furthermore, the values listed in Table 2 were used.

Table 1 Element Values of Simulations

Parameter	Value
R_1	0.2 Ω
R_2	0.3 Ω
L_1	63.35 μ H
L_2	43.53 μ H
k	0.116161
$\omega (=2\pi f)$	534071
Q_1	169.17
Q_2	77.49
L_m	6.01 μ H
C_1	56.10 nF
C_2	81.64 nF

$$R_L = Q_2 R_2 \sqrt{\frac{1 + Q_2^2}{(1 - k^2)^2 + k^4 Q_2^2 + k^2 Q_1 Q_2}} \quad (29)$$

Table 2 Element Values of Fig. 17 and Fig. 18

Parameter	Value
R_1	0.095 Ω
R_2	0.071 Ω
R_L	149.87 Ω
L_1	63.885 μ H
L_2	42.251 μ H
k	0.111472
$\omega (=2\pi f)$	534071
Q_1	359.15
Q_2	317.82
L_m	5.79 μ H
C_1	58.952 nF
C_2	84.754 nF

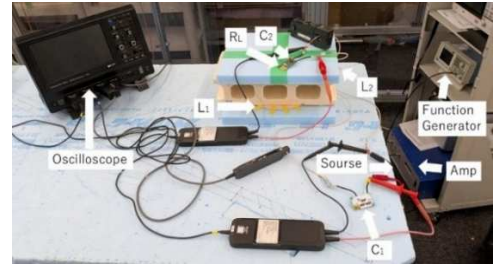


Fig. 6 Field test setup (Circuit of Fig. 5)

B. Simulation Results

Mathematica was used for analytical calculations, and MATLAB Simulink was used for electrical circuit simulations. The results obtained from the calculation of (28) using Mathematica and from the simulation of the circuit in Fig. 4 using MATLAB Simulink are presented in Fig. 7 through 16.

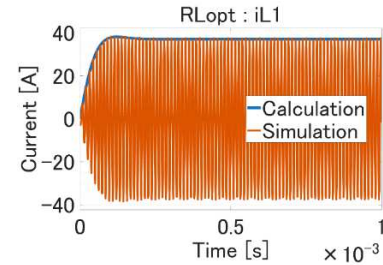


Fig. 7 Results of P-P Circuit Transmission Current (Optimal Load)

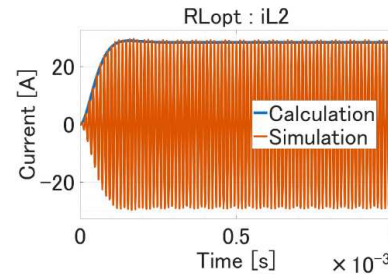


Fig. 8 Results of P-P Circuit Power Receiving Current (Optimal Load)

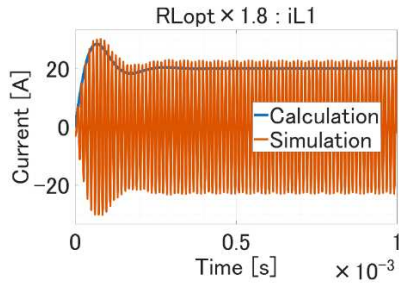


Fig. 9 Results of P-P Circuit Transmission Current (Optimal Load×1.8)

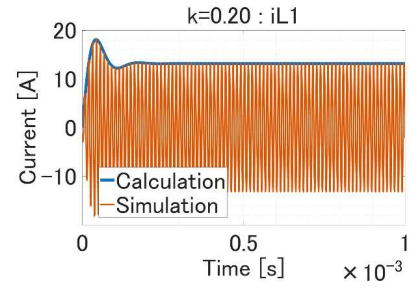


Fig. 13 Results of P-P Circuit Transmission Current (k=0.20)

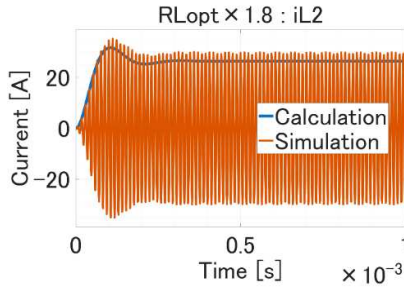


Fig. 10 Results of P-P Circuit Power Receiving Current (Optimal Load×1.8)

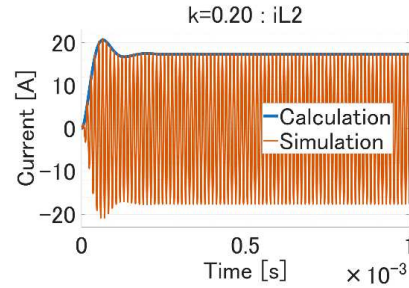


Fig. 14 Results of P-P Circuit Power Receiving Current (k=0.20)

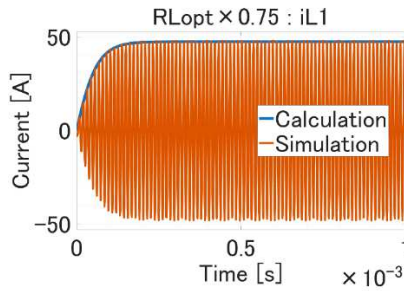


Fig. 11 Results of P-P Circuit Transmission Current (Optimal Load×0.75)

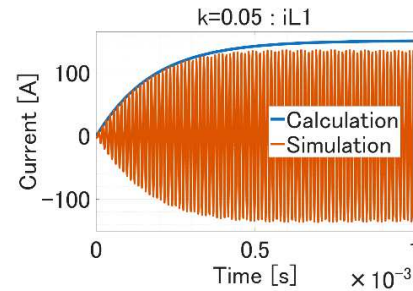


Fig. 15 Results of P-P Circuit Transmission Current (k=0.05)

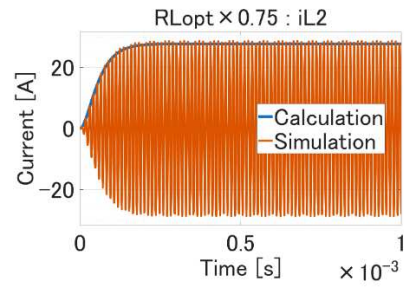


Fig. 12 Results of P-P Circuit Power Receiving Current (Optimal Load×0.75)

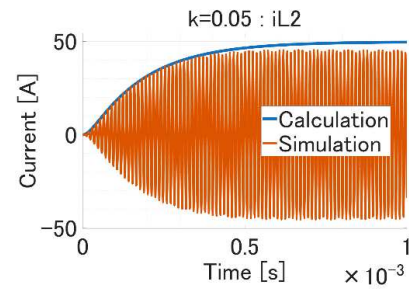


Fig. 16 Results of P-P Circuit Power Receiving Current (k=0.05)

At the optimal load, the error between the simulation and the analytical results is negligible. When the load deviates from the optimal condition, the error gradually increases. Even when the coupling coefficient is large, the derived equations maintain high accuracy. On the other hand, a resonance shift occurs when the coupling coefficient deviates from the value of 0.116161 listed in Table 1. However, the variation range of the coupling

coefficient in dynamic wireless power transfer (DWPT) during motion is approximately 0.2. Furthermore, as shown in Fig. 15 and Fig. 16, the error remains small during the initial stage of power transmission and reception. Therefore, the proposed method is considered to be applicable to DWPT systems.

C. Experimental Results

Experimental results obtained from the setup shown in Fig. 6 using the parameters listed in Table 2 are presented in Fig. 17 and Fig. 18.

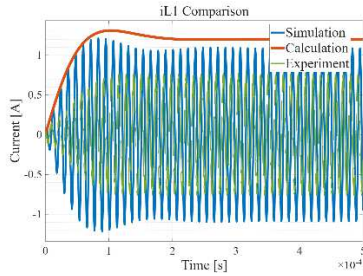


Fig. 17 Results of P-P Circuit Transmission Current (Experiment)

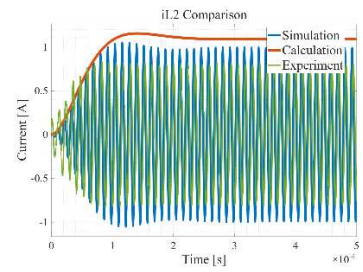


Fig. 18 Results of P-P Circuit Power Receiving Current (Experiment)

Here, noise attributed to the function generator was observed before the onset of the transient response in Fig. 17 and Fig. 18, and was therefore excluded from the experimental results. In the experimental tests, although the capacitor values deviated from the resonant values by 3.38 nF on the transmitter side and 0.73 nF on the receiver side, the overall waveform shape remained similar, thereby demonstrating the usefulness of the proposed model.

V. CONCLUSION

This paper was inspired by previous research (the Energy-Based Modeling (EBM) method) that focuses on energy, and by treating energy as a mediating variable, we successfully developed an envelope approximation model with reduced order for the P-P circuit in IPT systems. For the optimal load case, the

steady-state relative errors were as low as 1.3% for both the transmission-side current and the reception-side current. Furthermore, when the coupling coefficient is varied, a resonance shift occurs because the capacitor values are kept constant. However, since the proposed model can accurately capture the envelope during the initial transient period, it can compensate for the inherent weaknesses of the P-P circuit and is expected to be applicable to dynamic wireless power transfer (DWPT). In addition, this model is expected to improve response speed in control applications, such as feedforward control. By applying this theory, it is anticipated that envelope approximation models can also be developed for more complex circuits.

REFERENCES

- [1] Y. Wei, F. Wu and H. Liu, "Indirect Control Strategy of Secondary Charging Voltage and Current and Transient Analysis of LCC-S WPT System," in *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 11, no. 2, pp. 2364-2376, April 2023.
- [2] H. M. Hashmi, A. A. Shah and C. T. Rim, "General Dynamic Gyrator Models for Transient Analysis of IPT," 2024 IEEE Wireless Power Technology Conference and Expo (WPTCE), Kyoto, Japan, 2024, pp. 150-153.
- [3] W. Shi, J. Dong, T. B. Soeiro, J. Deng, C. Riekerk and P. Bauer, "Continuous Reduced-Order Dynamic Model Based on Energy Balancing for Inductive Power Transfer Systems," in *IEEE Transactions on Power Electronics*, vol. 37, no. 8, pp. 9959-9971, Aug. 2022.
- [4] Yoshinori Akamine, Hiroshi Fujimoto(The University of Tokyo), "Study of Dual-Side Transient Shaping Pulse Density Modulation for Wireless Power Transfer Systems", 2025 IEE-Japan Industry Applications Society Conference 1-120.
- [5] X. Li, Y. Zhang, S. Chen, X. Zhang, and Y. Tang, "Small-signal models of resonant converter with consideration of different duty cycle control schemes," *IEEE Trans. Power Electron.*, vol. 36, no. 11, pp. 13234-13247, Nov. 2021.
- [6] Z. U. Zahid et al., "Modeling and control of series-series compensated inductive power transfer system," *IEEE J. Emerg. Sel. Topics Power Electron.*, vol. 3, no. 1, pp. 111-123, Mar. 2015.
- [7] Hirono Namiki, Takehiro Imura, Yoichi Hori, "Unified Theory of Non-Resonant and Resonant Circuits in Inductive Power Transfer and Capacitive Power Transfer" *IEEJ Journal of Industry Applications*, 2024 volume 13, Issue 6, Page 633-644.
- [8] Hirono Namiki, Takehiro Imura, Yoichi Hori, "Unified Theory of Series, Parallel and LCL/CLC Resonant Circuits in Inductive Power Transfer and Capacitive Power Transfer" *IEEJ Journal of Industry Applications*, 2024 volume 13, Issue 6, Page 655-668.
- [9] C. S. Johnson et al., "Quantifying Charge Sharing Loss in Switched Capacitor Inverters for Capacitive Power Transfer Applications," in *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 13, no. 1, pp. 1330-1338, Feb. 2025.
- [10] Y. Yoshioka, N. Sakumichi, and N. Osawa, *Fundamentals of Transient Phenomena*, 2nd ed. Tokyo, Japan: Morikita Publishing, 2017 (in Japanese).
- [11] M. Nishimaki, T. Mori, and T. Arai, *Fundamentals of Electrical Circuits*. Tokyo, Japan: Morikita Publishing, 2007 (in Japanese).